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Tut 1.

Q1 (i) non decreasing

(ii) non increasing

(iii) increasing

(iv) non increasing or non decreasing.

Q2 (i) $\lg 64 = \log_2 2^6$

$= 6$

(ii) $\lg 2^{1000} = \log_2 2^{1000}$

$= 1000$

$\lfloor x \rfloor$

Q3 (i) Floor value of 37.99 = 37

Ceiling value of 37.99 = 38

$\lceil x \rceil$

(ii) $\lfloor \frac{10}{3} \rfloor = 3$

$\lceil \frac{10}{3} \rceil = 4$

Q4 $n! = n(n-1)!$ → True

$n \times n-1 \times n-2 \times n-3 \ldots = n \times (n-1)!$ //

Q5 Prove $\begin{bmatrix} n \\ r \end{bmatrix} = \begin{pmatrix} n \\ n-r \end{pmatrix}$

$\begin{bmatrix} n \\ r \end{bmatrix} = \frac{n!}{r!(n-r)!}$ $\begin{bmatrix} n \\ n-r \end{bmatrix} = \frac{n!}{(n-(n-r))!(n-r)!}$

$r = n-(n-r)$

$r! = [n-(n-r)]!$

$\begin{bmatrix} n \\ r \end{bmatrix} = \frac{n!}{[n-(n-r)]!\,(n-r)!} = \begin{bmatrix} n \\ n-r \end{bmatrix}$

Q6 $1+2+3+\ldots(k-1)$ for $k \geq 2$ ← arithmetic series $\underset{\text{1st term}}{a} + (n-1)\underset{\text{constant difference}}{d}$

Formula → $\frac{n}{2}[2a+(n-1)d]$

$a = 1$, $d = 1$ special case → $\frac{\overset{\text{last}}{n}}{2}[2+(\overset{\text{last 2nd}}{n-1})]$

$\frac{k-1}{2}[2+(k-2)]$

$= \frac{k(k-1)}{2}$

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Q7 $1 + 2 + 2^2 + \dots + 2^{i-1}$ ← Geometric series $\sum_{k=0}^{n} ar^k = \frac{a(r^{n+1}-1)}{r-1}$

$a = 1$, $r = 2$

$\sum_{k=0}^{i-1} \frac{2^{i-1+1}-1}{1}$

$= 2^i - 1$ //

Q8 (i) $n = 1$ LHS $1(1!) = 1$

RHS $(1+1)! - 1 = 2 \times 1 - 1 = 1$ ✓ LHS = RHS

$n = k$ where $\sum_{i=1}^{k} k(k!) = (k+1)! - 1$ ✓

$k+1 =$ where $\sum_{i=1}^{k+1} k(k!) = \boxed{(k+2)! - 1}$ → Prove this is right.

Proving

$\sum_{i=1}^{k+1} k(k!) = \sum_{i=1}^{k} k(k!) + (k+1)(k+1)!$

$= (k+1)! - 1 + (k+1)(k+1)!$ $(k+1)! [1 + (k+1)!] - 1$

$= (k+1)! [1 + (k+1)] - 1$

$= (k+1)! (1 + k + 1) - 1$

$= \underbrace{(k+1)! (k+2)}$ $- 1$

$= (k+2)! - 1$ (Proven)

(ii) $n = 1$ LHS $(1+x)^1 = 1 + x$

RHS $1 + x = 1 + x$ → LHS $\geq$ RHS proven

$n = k$ when we assume $(1+x)^k \geq 1 + kx$ to be true

where $x \geq -1$

$(1+x)^{k+1} \geq 1 + (k+1)x$ prove that this is true

$(1+x)^{k+1} = (1+x)^k \times (1+x)$

$\geq 1 + kx \times (1+x)$

$= 1 + x + kx + kx^2$

$= x[1+k] + \boxed{kx^2} + 1$ always positive value.

$> 1 + (k+1)x$ proven